Journal of Applied Mechanics and Technical Physics, Vol. 40, No. 4, 1999

SIMULATION OF "BELT"-TYPE GALATHEA PLASMA CONFIGURATIONS

A. V. Belimov, G. I. Dudnikova,
M. P. Fedoruk, and É. P. Shurina

UDC 533.951

Numerical models describing the formation of equilibrium "Belt"-type plasma configurations using direct discharge are developed. The magnetobaric characteristics $p(\Psi)$ of these configurations are determined. The calculation results are in agreement with experimental data.

Introduction. As is known, in the traditional plasma gaps studied in connection with the problem of controlled thermonuclear fusion, magnetic fields are produced using only coils resting on ground. From the practical viewpoint, of greatest interest are gaps in which the magnetic field in the plasma volume is absent or it has low intensity compared to the field in the magnetic barrier around the plasma [1]. However, slit-free magnetic configurations of this type cannot be produced using coils resting on ground. At the same time, the required configurations can be easily created by means of current-carrying conductors immersed in a plasma. In [1], such gaps are called "galatheas" and the conductors immersed in the plasma are called "myxines."

It should be noted that although galathea configurations have apparent advantages from the viewpoint of plasma confinement and very perfect experimental facilities have been designed, studies of these plasma configurations are at an early stage. The main reason why galatheas have not been adequately studied is the generally accepted opinion that galatheas are not promising as the basis of commercial thermonuclear reactors because of great technological difficulties. However, this opinion is wrong [1, 2].

Recently, a number of experimental and theoretical studies of galatheas have been performed (see [1–13] and the references in them). Thus, galatheas have been studied experimentally by the group of A. G. Frank on a setup galathea of the "Belt" type. A scheme of this gap was proposed by Morozov and Frank [3]. The gap is a toroidal quadrupole formed by two myxines between which an azimuthal current flows in a plasma. At present, straight "Belt" configurations have been studied experimentally in the regime of electrode discharge [3, 4]. In this case, the gap is a quartz chamber 100 cm long and 18 cm in diameter. The myxines are metal rods 95 cm long and 2 cm in diameter coated with an insulating layer 0.2 cm thick; the axes of the myxines are at a distance of $a = \pm 4.5$ cm from the axis of the chamber, and the electric currents in them have identical directions and magnitudes. The reverse current lead for the myxine current is also made in the form of two conductors that are parallel to the axis and located outside the vacuum chamber at a distance $y_0 = \pm 11.25$ cm from its axis. The electric currents in the outside conductors (hold elements) change the magnetic-field structure and decrease the force of attraction of the myxines. The gas (Ar or He) filling the chamber is previously ionized by a powerful ultraviolet lamp. Two plate grid electrodes are introduced into the chamber from both sides. A pulse voltage from a capacitor is applied to them, leading to further ionization of the gas and causing a current to flow in the plasma [4].

Proceeding to a discussion of the theoretical papers devoted to galatheas, we first consider stationary models for calculating static configurations. Morozov and Frank [3] constructed very general solutions of the Grad–Shafranov equation for the magnetobaric characteristics $p(\Psi)$ in the form of linear and quadratic splines. Morozov and Murzina [5] examined the equilibrium configurations of galatheas assuming that the

Institute of Computational Technologies, Siberian Division, Russian Academy of Sciences, Novosibirsk 630090. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 40, No. 4, pp. 3–10, July–August, 1999. Original article submitted November 14, 1997.

dependence $p(\Psi)$ is described by a linear spline. Savel'ev [6] constructed equilibrium configurations using conformal mappings for the case of infinitely thin "plasma–field" transition layers.

In [7, 8], the equilibrium plasma configurations in a magnetic field are calculated using the equations of stationary one-fluid magnetohydrodynamics. Brushlinskii et al. [7] solved this system of equations numerically by the finite-difference method in studying the plasma configurations in a galathea stellarator. The model proposed by Maikov et al. [8] makes it possible to take into account not only the plasma equilibrium but also the equilibrium of myxines in a magnetic field. Morozov [9] used the equations of stationary two-fluid magnetohydrodynamics for analysis of plasma shells of myxines.

The main disadvantage of the stationary approach is the *a priori* specification of the magnetobaric characteristic $p(\Psi)$, which is the plasma distribution between the magnetic surfaces. To obtain adequate information on this function and to study the corresponding magnetoplasma configurations and the dynamics of their formation, it is necessary to use nonstationary MHD models. At present, development of these models is beginning. Thus, Brushlinskii et al. [10, 11] performed a numerical simulation of the formation of a "Belt" configuration in the regime of direct discharge. They used the following simplified model: the myxines were assumed to be penetrable for the plasma and the current in them was constant. Therefore, such calculations could explain the plasma dynamics in the initial stage of the process. These calculations also supported the presumable scheme of formation of the "Belt" configuration. Dudnikova et al. [12, 13] developed MHD models of the formation of the "Belt" configuration taking into account the impenetrability of myxines and using the equations of one-fluid magnetohydrodynamics to describe the plasma dynamics. The Poisson's equation for the vector potential of the magnetic field was solved inside the myxine volume. The myxine's field was joined in a continuous manner with the plasma field. This model is described in detail in Sec. 2.

Numerical simulation plays an important role in solving the problem of galatheas. Indeed, even stationary models require invoking numerical methods to allow for the real geometry and design features of gaps. This is especially true for nonstationary models. In this connection, development of adequate mathematical models and effective numerical algorithms for implementing them remains very important. In the present paper, we propose two such models. A stationary model for a galathea gap of the "Belt" type is developed in Sec. 1. The algorithm for implementing this model is based on the finite-difference method. An MHD model for the "Belt" formation based on the equations of nonstationary one-fluid magnetohydrodynamics is developed in Sec. 2. Numerical implementation of this model is performed by a finite-difference method.

1. Stationary Model of the "Belt" Configuration. The stationary model of the "Belt" configuration is based on the equations of stationary one-fluid magnetohydrodynamics:

$$\nabla p = \frac{1}{c}(\boldsymbol{j}^{\rm pl} \times \boldsymbol{H}), \qquad \operatorname{rot} \boldsymbol{H} = \frac{4\pi}{c}(\boldsymbol{j}^{\rm pl} + \boldsymbol{j}^{\rm ex}), \qquad \operatorname{div} \boldsymbol{H} = 0. \tag{1.1}$$

Here $\boldsymbol{j}^{\rm pl}$ is the density of the current induced in the plasma, $\boldsymbol{j}^{\rm ex}$ is the density of the specified external current, p is the plasma pressure, and $\boldsymbol{H}$ is the magnetic-intensity vector.

Problem (1.1) is studied in a two-dimensional formulation since the myxine current has only the z component and the length of the setup along the z axis is much greater than its transverse dimensions and, hence, the edge effects can be ignored. Taking into account the symmetry conditions, we consider only a quarter of the initial flow. On the symmetry axes for the magnetic-field intensity $\boldsymbol{H}$, the plasma pressure p, and the plasma current density $\boldsymbol{j}^{\rm pl}$ the following boundary conditions are imposed:

$$\frac{\partial \boldsymbol{H}}{\partial \boldsymbol{n}}\Big|_{S_2} = 0. \qquad \frac{\partial p}{\partial \boldsymbol{n}}\Big|_{S_2} = 0, \qquad \frac{\partial \boldsymbol{j}^{\rm pl}}{\partial \boldsymbol{n}}\Big|_{S_2} = 0.$$

Here $\boldsymbol{n}$ is an outer normal to the boundary S_2. In addition, for the field intensity, the absence of a field away from the sources was specified (at a distance of about 10 radii of the chamber), $\boldsymbol{H}\big|_{S_1} = 0$. At the walls of the vacuum chamber and on the myxine surface, the nonpenetration conditions were imposed. Because the problem is two-dimensional, it is convenient to replace the magnetic-field intensity by the vector potential $\boldsymbol{A}$ ($\boldsymbol{H} = \operatorname{rot} \boldsymbol{A}$), which in this case has just one nonzero component $\boldsymbol{A} = (0, 0, \Psi)$ (following the tradition adopted in the theory of the Grad–Shafranov equations A_z is denoted by Ψ). Then, system (1.1) takes the form

$$\Delta\Psi = -\frac{4\pi}{c}j_z, \qquad \frac{dp}{dx} = -\frac{1}{c}j_z^{\rm pl}\frac{\partial\Psi}{\partial x}, \qquad \frac{dp}{dy} = -\frac{1}{c}j_z^{\rm pl}\frac{\partial\Psi}{\partial y}, \tag{1.2}$$

where $j_z \equiv j_z^{\rm pl} + j_z^{\rm ex}$.

The boundary conditions are given by

$$\Psi\Big|_{S_1} = 0, \quad \frac{\partial\Psi}{\partial \boldsymbol{n}}\Big|_{S_2} = 0, \quad \frac{\partial p}{\partial \boldsymbol{n}}\Big|_{S_2} = 0, \quad \frac{\partial j_z}{\partial \boldsymbol{n}}\Big|_{S_2} = 0. \tag{1.3}$$

A distinguishing feature of the problem is the presence of a free boundary of the plasma. In this connection, system (1.2) should be supplemented by a closure condition from which the boundary of the plasma volume can be determined. As this condition, we can use the dependence $p(\Psi)$, i.e., the magnetobaric characteristic. In the calculations, we used the linear magnetobaric characteristic

$$p(\Psi) = \begin{cases} 0, & \Psi < \Psi_1 < 0, \\ p_0(1 - \Psi/\Psi_*), & \Psi_1 < \Psi < \Psi_*, \\ 0, & \Psi \geqslant \Psi_* > 0 \end{cases} \tag{1.4}$$

and the quadratic characteristic [3, 5]

$$p(\Psi) = \begin{cases} 0, & \Psi < \Psi_1 < 0, \\ p_0((\Psi_* - \Psi)/\Psi_*)^2, & \Psi_1 < \Psi < \Psi_*, \\ 0, & \Psi \geqslant \Psi_* > 0, \end{cases} \tag{1.5}$$

where Ψ_* is the stream-function value on the boundary between the plasma and the outer magnetic shell and Ψ_1 is the current-function value on the boundary between the plasma and the magnetic shell of the myxines.

The self-conjugation of the Laplacian in the Poisson's equation for the potential allows one to obtain the variational formulation of the finite-element method in the Ritz form: finding a solution of the Poisson's equation is equivalent to minimizing the functional

$$I(v) = \int\limits_\Omega \nabla^T v \nabla v \, d\Omega - 2\int\limits_\Omega j_z v \, d\Omega \tag{1.6}$$

on the class of functions $v \in H_0^1(\Omega)$, where $H_0^1(\Omega)$ is the space of functions that are defined in the space Ω, have first-order derivatives, and satisfy zeroth first boundary conditions (1.3)

The relation $j_z = c(dp(\Psi)/d\Psi)$ allows one to obtain an analytic dependence for the plasma current from the known $p(\Psi)$. Thus, for the linear dependence (1.4) we have

$$j_z^{\rm pl} = -c\frac{p_0}{\Psi_*}, \tag{1.7}$$

and for the quadratic dependence (1.5), the current density in the region $\Psi_1 < \Psi < \Psi_*$ is

$$j_z^{\rm pl} = -2c\frac{p_0}{\Psi_*^2}(\Psi_* - \Psi). \tag{1.8}$$

The simulation region was approximated by irregular triangular finite elements. Minimization of the discrete analog of functional (1.6) was reduced to solving a system of linear algebraic equations by the conjugate-gradient method adapted to matrix representation in a rarefied row format [14]. Because a feature of the problem is the presence of a free boundary of the plasma, whose position is constantly refined, an error arises that is due to the rough approximation of the plasma boundary by finite element edges. This required adapting the finite-element grid to the current location of the plasma-volume boundaries. The following numerical algorithm was developed:

(1) specify the geometry of the simulation region and divide the region into finite elements (triangulation);

(2) specify the coefficients p_0, Ψ_*, and Ψ_1 of the magnetobaric characteristic $p(\Psi)$ and the current distribution $j^{\rm ex}$ in the myxines;

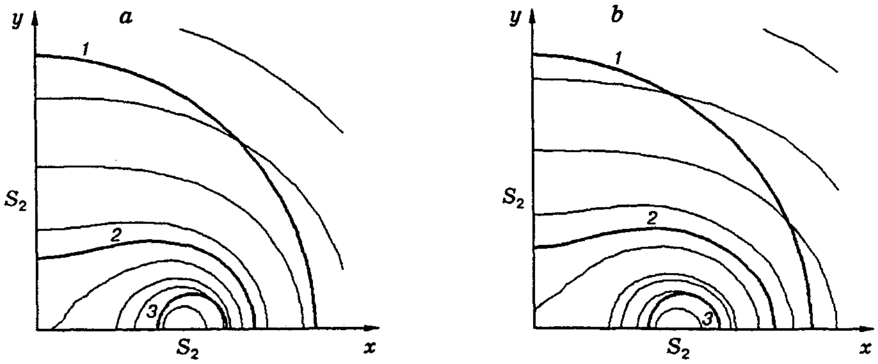

Fig. 1

(3) determine the initial region occupied by the plasma (for example, the entire accessible region up to the walls of the chamber and myxines);

(4) determine the initial distribution of the plasma current density $j^{\rm pl}$ (for example, uniform or zero);

(5) by minimizing the functional (1.6), determine the distribution of the vector potential Ψ;

(6) determine the plasma region $\Psi_1 \leqslant \Psi \leqslant \Psi_*$ in the vacuum chamber;

(7) adapt the grid by aligning the edges of the triangles nearest to the plasma boundary with the isolines $\Psi = \Psi_1$ and $\Psi = \Psi_*$;

(8) calculate the distributions of the plasma pressure p and the plasma current density $j^{\rm pl}$ from formulas (1.4), (1.5) and (1.7), (1.8), respectively.

(9) the conditions of termination of the iterative process is as follows: if the residual difference norm in two neighboring iterations is smaller than the specified accuracy ε, the iterative process is terminated; otherwise, the iterative procedure is continued from item 5 taking into account the changed geometry of the plasma volume.

Figure 1a and b shows the magnetic field lines for the cases of linear and quadratic magnetobaric characteristics, respectively. Curve 1 in Fig. 1 shows the boundary of the chamber, curve 2 shows the boundary of the plasma volume, and curve 3 shows the boundary of the myxine.

2. Nonstationary Model of the "Belt" Configuration The nonstationary model of the "Belt" configuration is based on the following equations of one-fluid magnetohydrodynamics:

$$\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \boldsymbol{u}) = 0, \quad \rho\left(\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\nabla)\boldsymbol{u}\right) = -\nabla p - \frac{1}{4\pi}[\Delta \boldsymbol{A} \times \operatorname{rot}\boldsymbol{A}], \tag{2.1}$$

$$\frac{\partial \boldsymbol{A}}{\partial t} = [\boldsymbol{u} \times \operatorname{rot}\boldsymbol{A}] + \nu_m \Delta \boldsymbol{A}, \quad \frac{\partial p}{\partial t} + (\boldsymbol{u}\nabla)p = -\gamma p \operatorname{div}\boldsymbol{u} + \frac{\gamma - 1}{4\pi}\nu_m(\Delta \boldsymbol{A})^2 + \chi(\gamma - 1)\Delta T.$$

Here ρ is the plasma density, $\boldsymbol{u}$ is its mass average velocity, p and T are the gas-dynamic pressure and temperature of the plasma, respectively, $\boldsymbol{A}$ is the vector potential, $\nu_m = c^2/(4\pi\sigma)$ is the magnetic viscosity, σ is the plasma conductivity, and χ is the thermal conductivity. Taking into account the experimental conditions of [4], we use the following characteristic parameters:

— magnetic field $H_0 = 1$ kG;

— length $L = 10$ cm;

— characteristic plasma concentration $n_0 = 10^{15}$ cm^{-3};

— velocity $V_A^0 \simeq H_0/(4\pi\rho_0)^{1/2} \simeq 3.5 \cdot 10^6$ cm/sec;

— characteristic time $t_0 = L/V_A^0 \simeq 2.9 \cdot 10^{-6}$ sec;

— pressure $p_0 = H_0^2/(8\pi) \simeq 4 \cdot 10^4$ dyn/cm^2;

— temperature $T_0 \simeq 25$ eV;

— current density $j_0 = cH_0/(4\pi L) \simeq 80$ A/cm^2.

In the calculations below, the thermal conductivity χ was assumed to be equal to zero.

As in Sec. 1, we consider two-dimensional plasma flows in the plane (x, y) that are symmetric about the axes $x = 0$ and $y = 0$. In this case, just one component of the vector potential $\boldsymbol{A} = (0, 0, \Psi)$ is different from zero. In view of the symmetry of the problem, we consider only the first quadrant $0 \leqslant x \leqslant x_m$, $0 \leqslant y \leqslant y_m$ (in dimensionless variables).

The boundary conditions on the lines $x = 0$ and $y = 0$ are the natural conditions of flow symmetry. On the external boundaries of the calculation region Γ, an electric field is specified in the form of the following dependence of Ψ on t:

$$\Psi(t) = \begin{cases} A_1 \sin(\omega t), & \omega t \leqslant \pi/2, \\ A_1, & \omega t > \pi/2. \end{cases} \tag{2.2}$$

Here A_1 and ω are dimensionless parameters that characterize the magnitude and rate of increase in the vector potential. It is assumed that the myxine occupies the region $x_0 \leqslant x \leqslant x_1$, $y_0 \leqslant y \leqslant y_1$. In this case the dimension of the myxine is 1/5–1/10 of the total length of the calculation region in each direction.

Inside the myxine volume, we solve the Poisson's equation

$$\Delta\Psi = -\frac{4\pi}{c} j_z(x, y, t). \tag{2.3}$$

The value of j_z in the myxine is given by the "Ohm law" $j_z = \sigma(E_z + \mathcal{E})$, where $\mathcal{E}$ is the e.m.f. characteristic. In the present calculations, it was assumed that $j_z = j_0 = \text{const}$. The field inside the myxine is joined by a conventional procedure with the plasma field $[H_n] = 0$. The impenetrability condition $u_n = 0$ is imposed on the myxine surface. The symmetry conditions $\partial f / \partial \boldsymbol{n} = 0$ are specified for the remaining sought functions on the myxine surface.

At the initial time $t = 0$, we find the vector-potential distribution by solving Eq. (2.3). In this case, $j = j_0$ inside the myxine volume and $j = 0$ outside the myxine volume. The conditions on the boundaries of the calculation region for (2.3) at the initial time are formulated as follows:

$$\Psi = 0 \quad \text{for } x = x_m,\ y = y_m, \qquad \frac{\partial \Psi}{\partial x} = 0 \quad \text{for } x = 0, \qquad \frac{\partial \Psi}{\partial y} = 0 \quad \text{for } y = 0.$$

Then, the region is filled with a fully ionized plasma with $n = n_0(x, y)$ and $T = T_0(x, y)$. In the present calculations, the initial distributions of the plasma density and temperature were constant: $n_0(x, y) = n_0$ and $T_0(x, y) = T_0$. On the outer boundaries $x = x_m$ and $y = y_m$, the distribution of the potential Ψ was specified according to (2.2). The myxine current was equal to $j_0 = \text{const}$, and its direction coincided with the current direction on the outer boundary (the α regime) or was opposite to it (the β regime).

Equations (2.1) were solved using a difference scheme with oriented differences, and the Poisson's equations (2.3) were solved by the method of successive overrelaxation with acceleration. Typical calculation variants contained 50×50 or 100×100 calculation cells, so that 10 to 20 calculation cells in each direction fell in the myxine region. To check the calculation accuracy, we verified the law of conservation of the total energy of the system, which was satisfied with accuracy up to several percent.

In [12, 13], the following regimes of formation of quasistationary configurations are considered: the basic α regime, the basic β regime, and the α regime with varying currents in the myxines (the $\alpha\mu$ regime). In the basic regimes, the myxine currents are considered constant, and they correspond to the "basic solutions" of the Grad–Shafranov equation described in [3]. These calculations were performed for myxines of various dimensions and various initial plasma parameters and boundary conditions. The results obtained are qualitatively similar, and, hence, in the present paper, we give only one example for each of the basic α and β regimes.

2.1. *Basic α Regime.* Figure 2a and b shows [for $n_0 = 1$, $T_0 = 0.2$, $j_0 = -80$, and $\tilde{\nu} \equiv c^2/(4\pi\sigma L V_A) = 10^{-2}$] patterns of the magnetic field lines (the dashed curve corresponds to a value $\Psi = 0$) and pressure isolines, respectively, in the quasistationary regime.

A characteristic feature of the configuration formed is that plasma contacts the myxine surface. This is natural for the nearly zeroth conductivity of the myxine since the current in it is considered constant. In addition, the plasma distribution is uniform at the initial time. We note that in the experiments of A. G.

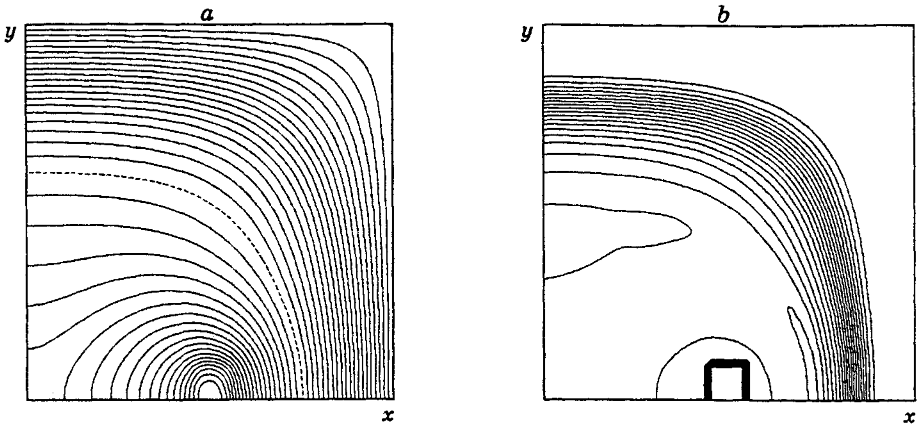

Fig. 2

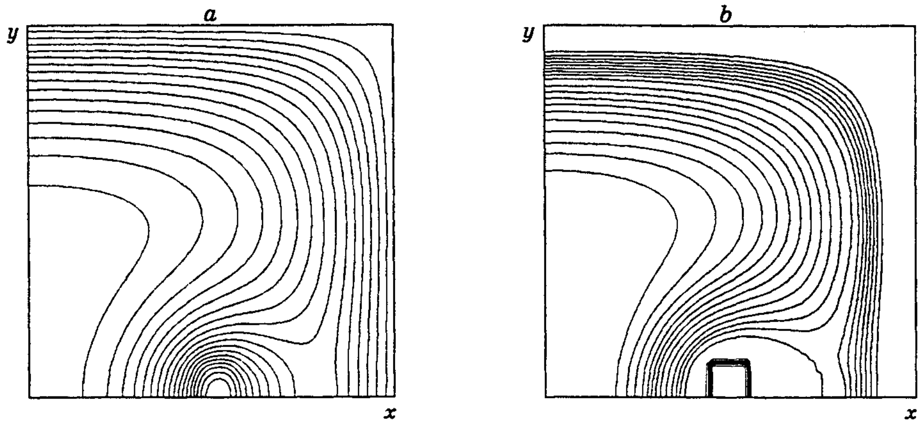

Fig. 3

Frank, the compressed plasma is also adjacent to the myxines [4]. Contact of the plasma with the myxines could apparently be eliminated if the plasma density at the initial time is set equal to zero over a rather wide region around the myxine and the plasma conductivity is assumed to be rather high. Furthermore, the myxine conductivity should also be rather high in order that a thin skin layer be formed in it.

From the numerical calculation of [12, 13] it follows that the magnetobaric characteristic $p(\Psi)$ constructed on the basis of topograms for Ψ and p is with high accuracy a linear spline similar to the one used in [3, 5] in determining the basic solution of the Grad–Shafranov equation. The linearity of $p(\Psi)$ is obviously a result of the static character of the configuration at constant plasma conductivity.

2.2 *Basic β Regime.* By definition, in the β-regime the directions of the plasma and myxine currents are opposite. The calculation results for this regime (for $n_0 = 1$, $T_0 = 0.2$, $j_0 = 80$, and $\tilde{\nu} = 10^{-2}$) are presented in Fig. 3 [magnetic-field lines (a) and plasma-pressure isolines (b)]. In this case too, transition to static configurations with a magnetobaric characteristic in the form of a linear spline is observed. Naturally, the β configuration calculated by the iterative method is similar to the configuration obtained from the static Grad–Shafranov equation [5]. Finally, we note that the main feature of the β configuration — separation of the plasma from the myxine — is also clearly manifested in experiments.

Conclusion. In the present paper, we developed stationary and nonstationary models of a galathea gap of the "Belt" type. The results of numerical calculations using the nonstationary model show that the equilibrium (stationary) configurations of these gaps can be obtained by the iterative method. It is shown that the magnetobaric characteristics $p(\Psi)$ in the stationary "basic" α and β regimes at finite constant conductivity of the plasma are described by linear splines. The numerical calculations for the stationary and nonstationary models are in good agreement. The results obtained in the paper are qualitatively consistent with the experiments of [4].

We are grateful to A. G. Frank and A. I. Morozov for assistance in the present investigation and fruitful discussions of the results obtained.

This work was supported by the Russian Foundation for Fundamental Research (Grant No. 98-02-17115a).

REFERENCES

1. A. I. Morozov, "On galatheas — plasma gaps with conductors immersed in a plasma," *Fiz. Plazmy*, **18**, No. 3, 305-316 (1992).
2. A. I. Morozov and V. I. Khripunov, "Estimation of the parameters of an annular 'myxine' for thermonuclear galathea reactors," *Fiz. Plazmy*, **18**, No. 7, 838–849 (1992).
3. A. I. Morozov and A. G. Frank, "Toroidal multipole galathea gap with azimuthal current," *Fiz. Plazmy*, **11**, No. 3, 982–989 (1994).
4. S. Yu. Bogdanov, V. S. Markov, A. I. Morozov, and A. G. Frank, "Plasma configuration 'galathea–Belt.' The first results of experiments," *Pis'ma Zh. Tekh. Fiz.*, **21**, No. 24, 5–9 (1995).
5. A. I. Morozov and A. V. Murzina, "Simple equilibrium galathea configurations of the 'Belt' type," *Fiz. Plazmy*, **22**, No. 6, 551–563 (1996).
6. V. V. Savel'ev, "Equilibrium figures of an ideal plasma with a current in a magnetic field," *Fiz. Plazmy*, **21**, No. 3, 216–220 (1995).
7. K. V. Brushlinskii, N. M. Zueva, M. S. Mikhailova, et al., "Numerical simulation of straight screw cords with conductors immersed in a plasma," *Fiz. Plazmy*, **20**, No. 2, 284–292 (1994).
8. A. R. Maikov, A. I. Morozov, and A. G. Sveshnikov, "On the interaction of conductors in galathea gaps," *Fiz. Plazmy*, **22**, No. 1, 93–95 (1996).
9. A. I. Morozov and V. P. Pastukhov, "On the transition processes near closed current-carrying conductors (myxines) immersed in a plasma," *Fiz. Plazmy*, **18**, No. 6, 790–798 (1992).
10. K. V. Brushlinskii, K. P. Gorshenin, and A. I. Morozov, "Numerical simulation of the initial stage of formation of the 'Belt' plasma configuration," *Pis'ma Zh. Tekh. Fiz.*, **21**, No. 22, 67–70 (1995).
11. K. V. Brushlinskii and K. P. Gorshenin, "A planar MHD model for the formation of a plasma configuration with conductors immersed in it," *Mat Model.*, **9**, No. 5, 28–36 (1997).
12. G. I. Dudnikova, A. I. Morozov, and M. P. Fedoruk, "Formation of 'Belt'-type plasma galathea configurations," *Pis'ma Zh. Tekh. Fiz.*, **22**, No. 21, 45–51 (1996).
13. G. I. Dudnikova, A. I. Morozov, and M. P. Fedoruk, "Numerical simulation of straight 'Belt'-type galathea plasma configurations," *Fiz. Plazmy*, **23**, No. 5, 387–396 (1997).
14. J. M. Ortega, *Introduction to Parallel and Vector Solution of Linear Systems*, Plenum Press, New York–London (1988).